2014 Summer Review for Students Entering Pre-Calculus with Trigonometry

1. Using Function Notation and Identifying Domain and Range
2. Multiplying Polynomials and Solving Quadratics
3. Solving with Trig Ratios and Pythagorean Theorem
4. Multiplying and Dividing Rational Expressions
5. Adding and Subtracting Rational Expressions
6. Finding the Equation for the Inverse of a Function
7. Completing the Square to Find the Vertex of a Parabola

TI-84 Plus Graphing Calculator is required for this course.

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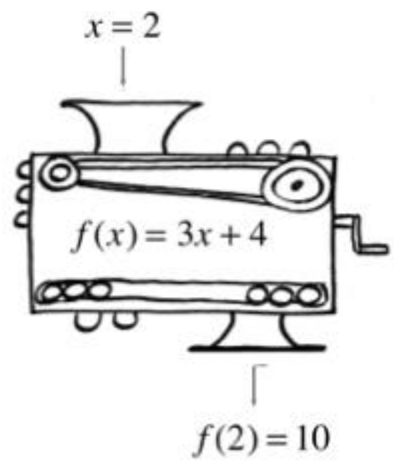
TOPIC 1: Using Function Notation and Identifying Domain and Range

An equation is called a function if there exists no more than one output for each input. If an equation has two or more outputs for a single input value, it is not a function. The set of possible inputs of a function is called the domain, while the set of all possible outputs of a function is called the range.

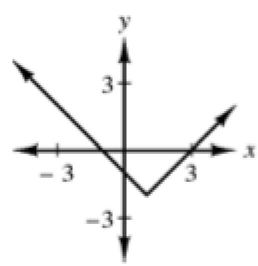
Functions are often given names, most commonly "*f*," "*g*," or "*h*." The notation $f(x)$ represents the output of a function, named *f*, when *x* is the input. It is pronounced "*f* of *x*." The notation $g(2)$, pronounced "*g* of 2," represents the output of function *g* when $x = 2$.



Similarly, the function $y = 3x + 4$ and $f(x) = 3x + 4$ represent the same function. Notice that the notation is interchangeable, that is $y = f(x)$. In some textbooks, $3x + 4$ is called the rule of the function. The graph of $f(x) = 3x + 4$ is a line extending forever in both the *x* (horizontal) and the *y* (vertical) directions, so the domain and range of $f(x) = 3x + 4$ are all real numbers.

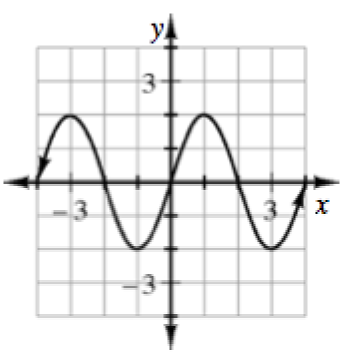
Example 1: $f(x) = |x - 1| - 2$

Solution:



We start by graphing the function, as shown at right. Since we can use any real number for *x* in this equation, the domain is all real numbers. The smallest possible result for *y* is –2, so the range is $y \geq -2$. By looking at the graph or substituting$x = 2$ into the equation, $f(2) = 2 - 1 - 2 = -1$. To solve $f(x) = 3$, find the points where the horizontal line $y = 3$ intersects the graph or solve the equation $3 = x - 1 - 2$, which yields $x = -4$ or $x = 6$.

Example 2: $f(x)$ is given by the graph below.



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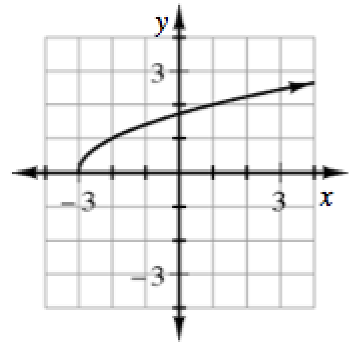
Example 3: $f(x)=\sqrt{x+3}$

Solution:

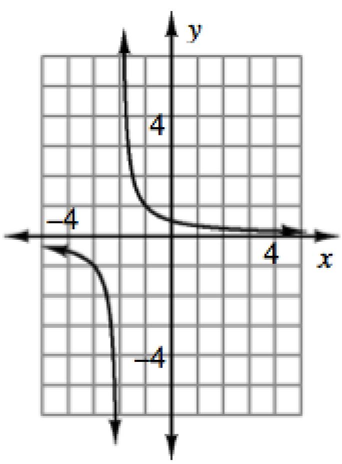


Again, we start by making a graph of the function, which is shown at right. Since the square root of a negative number does not exist, we can only use x-values of -3 or larger. Thus, the domain is $x \geq -3$. We can see from the graph and the equation that the smallest possible y-value is zero, so the range is $y \geq 0$. Looking at the graph gives an approximate answer when $x = 2$ of $y \approx 2.25$.

Or, by substituting $x = 2$ into the equation, we get $f(2)=\sqrt{2+3}=\sqrt{5}$. To solve $f(x) = 3$, find the point where $y = 3$ intersects the graph or solve $3=\sqrt{x+3}$, which gives $x = 6$.

Here are some more to try. Find the Domain and Range for 1-3.

1.



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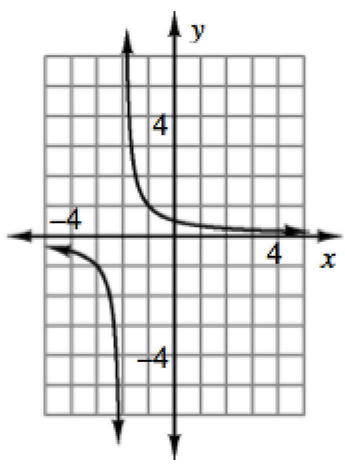

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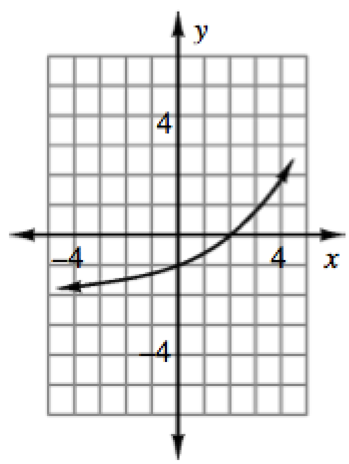

4. If $f(x)=3-x^2$, calculate $f(5)$ and $f(3a)$.

5. If $f(x)=x^2+5x+6$, solve $f(x)=0$

6. If $g(x)=5-3x^2$, calculate $g(-2)$ and $g(a+2)$.

Answers: 1. Domain: $x \neq -2$, Range: $y \neq 0$ 2. Domain: all real numbers, Range: $y \geq -5$

3. Domain: all real numbers, Range: $y > -2$

4. $f(5) = -22,\ f(3a) = 3 - 9a^2$

5. $x = -2$ or $x = -3$

6. $g(-2),\ g(a+2) = -3a^2 - 12a - 7$

TOPIC 2: Multiplying Polynomials and Solving Quadratics

Polynomials can be multiplied (changed from the area written as a product to the area written as a sum) by using the Distributive Property or generic rectangles.

Example 1: Multiply $-5x(-2x + y)$.

Solution: Using the Distributive Property $\underbrace{-5x(-2x + y)}_{\text{area as a product}} = -5x \bullet -2x + -5x \bullet y = \underbrace{10x^2 - 5xy}_{\text{area as a sum}}$

Example 2: Multiply $(x - 3)(2x + 1)$.

Solution: Although the Distributive Property may be used, for this problem and other more complicated ones, it is beneficial to use a generic rectangle to find all the parts.

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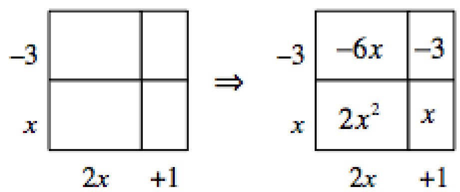


$\Rightarrow (x-3)(2x+1) = 2x^2 - 5x - 3$
area as a product   area as a sum

Solving quadratics first required factoring the polynomials to change the sum into a product. It is the reverse of multiplying polynomials and using a generic rectangle is helpful. Once the expression is factored, then the factors can be found using the Zero Product Property.

Example 3: Solve $x^2 + 7x + 12 = 0$

Solution: First, factor the polynomial. Sketch a generic rectangle with 4 sections.

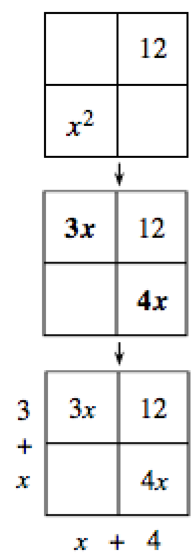
Write the x^2 and the 12 along one diagonal.

Find two terms whose product is $12 \bullet x^2 = 12x^2$ and whose sum is $7x$. That is, $3x$ and $4x$. This is the same as a Diamond Problem.

Write these terms as the other diagonal.

Find the base and height of the rectangle by using the partial areas.

Write the factored equation. $x^2 + 7x + 12 = (x + 3)(x + 4) = 0$



Then, using the Zero Product Property, we know that either $(x + 3)$ or $(x + 4)$ is equal to zero (since their product is zero). This means that $x = -3$ or $x = -4$.

Here are some more to try. Multiply the expression in problems 1 through 4 and solve the equation in problems 6 and 7.

1. $2x(x-1)$

2. $(3x+2)(2x+7)$

3. $(2x-1)(3x+1)$

4. $(x+y)(x+2)$

5. $x^2+5x+6=0$

6. $2x^2+5x+3=0$

1. $2x^2-2\text{x}$

2. $6x^2+25x+14$

3. $6x^2-x-1$

4. $x^2+xy+2x+2y$

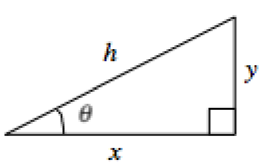
5. $x=-2,-3$

6. $x=-\frac{3}{2}, -1$

TOPIC 3: Solving With Trigonometric Ratios and the Pythagorean Theorem

Three trigonometric ratios and the Pythagorean Theorem can used to solve for the missing side lengths and angle measurements in any right triangle.

In the triangle below, when the sides are described relative to the angle θ, the opposite leg is y and the adjacent leg is x. The hypotenuse is h regardless of which acute angle is used.



$$\tan\theta = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{y}{x}$$

$$\sin\theta = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{y}{h}$$

$$\cos\theta = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{x}{h}$$

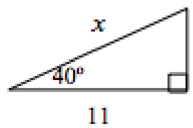
Also for the triangle above, from the Pythagorean Theorem: $h^2 = x^2 + y^2$.

Example 1: A rectangle has a diagonal of 16 cm and one side of 11 cm. What is the length of the other side?

Solution: The diagonal represents the hypotenuse of a right triangle and the one given side represents one of the legs. Using the Pythagorean Theorem:

$$h^2 = x^2 + y^2 \Rightarrow 16^2 = 11^2 + y^2 \Rightarrow 256 = 121 + y^2$$
$$135 = y^2 \Rightarrow y = \sqrt{135} \approx 11.62 \text{ cm}$$

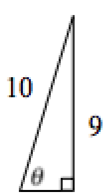
Example 2: Solve for *x* in the triangle at right.



Solution: Based on the 40º angle, 11 is the adjacent side and x is the hypotenuse. Use the cosine ratio to solve.

$$\cos 40^\circ = \frac{11}{x}$$
$$x \cos 40^\circ = 11 \Rightarrow x = \frac{11}{\cos 40^\circ} \approx 14.36 \text{ units}$$

Example 3: A ten-foot ladder is leaning against the side of a house. If the top of the ladder touches the house nine feet above the ground, what is the angle made by the ladder and the ground?



Solution: Make a diagram of the situation similar to the one at right. The ladder (10) is the hypotenuse and the house (9) is the opposite leg. Using the sine ratio, $\sin\theta = \frac{9}{10}$ To find θ, "undo" the sine function with the inverse sine function ($\sin^{-1} x$) as follows:

$$\sin\theta = \frac{9}{10}$$
$$\sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{9}{10}\right)$$
$$\theta = \sin^{-1}\frac{9}{10}$$
$$\theta \approx 64.2^\circ$$

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Here are some more to try. Use the right triangle trigonometric ratios or the Pythagorean Theorem to solve for the variable(s).

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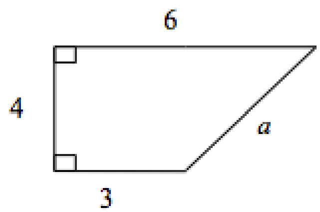

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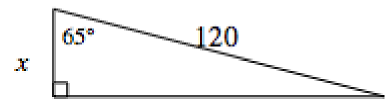

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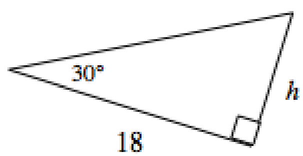

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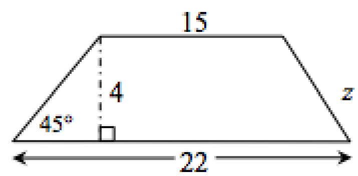

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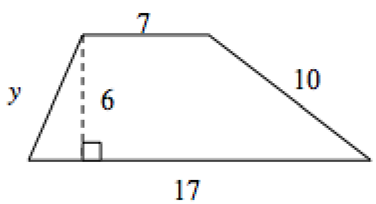

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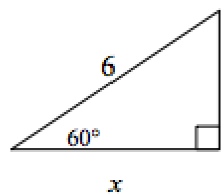

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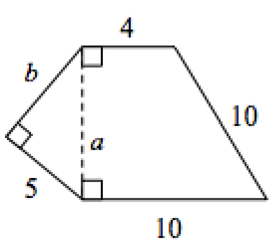

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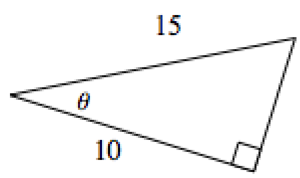

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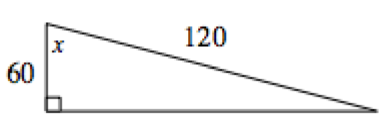

Draw a diagram and solve the following.

10. What is the distance between $(-6, -6)$ and $(-3, 2)$?

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Answers:

1. 5 units
2. ≈ 50.7 units
3. ≈ 10.4 units
4. 5 units
5. $\sqrt{40} \approx 6.32$ units
6. 3 units
7. $a = 8$, $b = \sqrt{39} \approx 6.24$ units
8. $\approx 48.2°$
9. $60°$
10. $\sqrt{73} \approx 8.5$ units

TOPIC 4: Multiplying and Dividing Rational Expressions

Multiplication or division of rational expressions follows the same procedure used with numerical fractions. However, it is often necessary to factor the polynomials in order to simplify them. Factors that are the same in the numerator and denominator are equal to 1. For example: $\frac{x^2}{x^2} = 1$, $\frac{(x+2)}{(x+2)} = 1$ and $\frac{(3x-2)}{(3x-2)} = 1$ but $\frac{5-x}{x-5} = \frac{-(x-5)}{x-5} = -1$.

When dividing rational expressions, change the problem to multiplication by inverting (flipping) the second expression (or any expression that follows a division sign) and completing the process as you do for multiplication.

In both cases, the simplification is only valid provided that the denominator is not equal to zero. See the examples below.

Example 1: Multiply $\frac{x^2 + 6x}{(x+6)^2} \cdot \frac{x^2 + 7x + 6}{x^2 - 1}$ and simplify the result.

Solution:

After factoring, the expression becomes: $\frac{x(x+6)}{(x+6)(x+6)} \cdot \frac{(x+6)(x+1)}{(x+1)(x-1)}$

After multiplying, reorder the factors: $\frac{(x+6)}{(x+6)} \cdot \frac{(x+6)}{(x+6)} \cdot \frac{x}{(x-1)} \cdot \frac{(x+1)}{(x+1)}$

Since $\frac{(x+6)}{(x+6)} = 1$ and $\frac{(x+1)}{(x+1)} = 1$, simplify: $1 \cdot 1 \cdot \frac{x}{x-1} \cdot 1 \;\Rightarrow\; \frac{x}{x-1}$ for $x \neq -6, -1$, or 1

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Here are some more to try. Multiply or divide each pair of rational expressions. Simplify the result. Assume the denominator is not equal to zero.

1. $\dfrac{x^2+5x+6}{x^2-4x}\cdot\dfrac{4x}{x+2}$

2. $\dfrac{x^2-2x}{x^2-4x+4}\div\dfrac{4x^2}{x-2}$

3. $\dfrac{x^2-16}{(x-4)^2}\cdot\dfrac{x^2-3x-18}{x^2-2x-24}$

4. $\dfrac{x^2-x-6}{x^2+3x-10}\cdot\dfrac{x^2+2x-15}{x^2-6x+9}$

Answers:

1. $\dfrac{4(x+3)}{x-4}$ 2. $\dfrac{1}{4x}$ 3. $\dfrac{x+3}{x-4}$ 4. $\dfrac{x+2}{x-2}$

TOPIC 5: Adding and Subtrating Rational Expressions

Addition and subtraction of rational expressions uses the same process as simple numerical fractions. First, if necessary find a common denominator. Second, convert the original fractions to equivalent ones with the common denominator. Third, add or subtract the new numerators over the common denominator. Finally, factor the numerator and denominator and simplify, if possible. Note that these steps are only valid provided that the denominator is not zero.

Example 1: $\dfrac{3}{2(n+2)}+\dfrac{3}{n(n+2)}$

The least common multiple of $2(n+2)$ and $n(n+2)$ is $2n(n+2)$. $\dfrac{3}{2(n+2)}+\dfrac{3}{n(n+2)}$

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To get a common denominator in the first fraction, multiply the fraction by $\frac{n}{n}$, a form of the number 1. Multiply the second fraction by $\frac{2}{2}$.

$$= \frac{3}{2(n+2)} \cdot \frac{n}{n} + \frac{3}{n(n+2)} \cdot \frac{2}{2}$$

Multiply the numerator and denominator of each term. It may be necessary to distribute the numerator.

$$= \frac{3n}{2n(n+2)} + \frac{6}{2n(n+2)}$$

Add, factor, and simplify the result. (Note: $n \neq 0$ or -2)

$$= \frac{3n+6}{2n(n+2)} \Rightarrow \frac{3(n+2)}{2n(n+2)} \Rightarrow \frac{3}{2n}$$

Example 2: $\mathbf{\frac{3}{x-1} - \frac{2}{x-2}}$

$$\frac{3}{x-1} - \frac{2}{x-2} \Rightarrow \frac{3}{x-1} \cdot \frac{x-2}{x-2} - \frac{2}{x-2} \cdot \frac{x-1}{x-1} \Rightarrow \frac{3x-6-2x+2}{(x-1)(x-2)} \Rightarrow \frac{x-4}{(x-1)(x-2)}$$

Here are a few more to try. Add or subtract each expression and simplify the result. In each case assume the denominator does not equal zero.

1. $\frac{2}{x+4} - \frac{x-4}{x^2-16}$

2. $\frac{3x}{2x^2-8x} + \frac{2}{x-4}$

3. $\frac{5x+9}{x^2-2x-3}+\frac{6}{x^2-7x+12}$

4. $\frac{x+4}{x^2-3x-28}-\frac{x-5}{x^2+2x-35}$

Answers:

1. $\frac{1}{x+4}$
2. $\frac{7}{2(x-4)}=\frac{7}{2x-8}$
3. $\frac{5(x+2)}{(x-4)(x+1)}=\frac{5x+10}{x^2-3x-4}$
4. $\frac{14}{(x-7)(x+7)}=\frac{14}{x^2-49}$

TOPIC 6: Finding the Equation for the Inverse of a Function

To find the equation for the inverse of a function, you can interchange the *x* and *y* variables and then solve for *y*. This also means that the coordinates of points that are on the graph of the function will be reversed on the graph of the inverse. Here are some examples:

Example 1: **Write the equation for the inverse of $y = 2(x + 3)$.**

Solution: We can interchange the *x* and the *y* to get the equation of the inverse. To get our final answer, we solve for *y*, as shown at right.

$$2(y+3)=x$$
$$(y+3)=\tfrac{x}{2}$$
$$y=\tfrac{x}{2}-3$$

Answer: $y=\frac{x}{2}-3$

Example 2: **Write the equation for the inverse of $y = -\frac{2}{3}x + 6$.**

Solution: Interchanging the *x* and the *y*, we get $x=-\frac{2}{3}y+6$. Solving for *y* gives $y=-\frac{3}{2}(x-6)=-\frac{3}{2}x+9$.

Answer: $y=-\frac{3}{2}x+9$

Here are some more to try. Find an equation for the inverse of each function.

1. $y = \frac{x+1}{4}$
2. $y = \frac{1}{3}x + 2$
3. $y = 1 + \sqrt{x+5}$
4. $y = x^3 + 1$

Answers:

1. $y = 4x - 1$
2. $y = \frac{x+2}{3}$
3. $y = (x-1)^2 - 5$
4. $y = \sqrt[3]{x-1}$

TOPIC 7: Completing the Square to Find the Vertex of a Parabola

If a quadratic function is in graphing form then the vertex can be found easily and a sketch of the graph can be made quickly. If the equation of the parabola is not in graphing form, the equation can be rewritten in graphing form by completing the square.

First, recall that $y = x^2$ is the parent equation for quadratic functions and the general equation can be written in graphing form as $y = a(x - h)^2 + k$ where (h, k) is the vertex, and relative to the parent graph the function has been:

- Vertically stretched, if the absolute value of a is greater than 1
- Vertically compressed, if the absolute value of a is less than 1
- Reflected across the x-axis, if a is less than 0

Example 1: **Complete the square to change $y = x^2 + 8x + 10$ into graphing form and name the vertex.**

Solution: Use an area model to make $x^2 + 8x$ into a perfect square. To do this, use half of the coefficient of the x-term on each side of the area model, and complete the upper right corner of the square, as shown below.

4	$4x$	16
x	x^2	$4x$
	x	4

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Your square shows that $(x + 4)^2 = x^2 + 8x + 16$. But your original expression is $x^2 + 8x + 10$, which is 6 fewer than $(x + 4)^2$. So you can write
$y = x^2 + 8x + 10 = (x+4)^2 - 6$.

Because the function is now in graphing form, $y = a(x - h)^2 + k$, you know the vertex is $(h, k) = (-4, -6)$.

Here are some more to try. Write each equation in graphing form. If needed, complete the square to do so. Then state the vertex, y-intercept, and the stretch factor and sketch a graph.

1. $y = x^2 - 6x + 9$
2. $y = x^2 + 3$
3. $y = x^2 + 2x - 3$
4. $y = -x^2 - 6x + 10$

Answers:

1. $y = (x - 3)^2$; (3, 0); (0, 9); $a = 1$
2. $y = (x - 0)^2 + 3$; (0, 3); (0, 3); $a = 1$
3. $y = (x + 1)^2 - 4$; (−1, −4); (0, −3); $a = 1$
4. $y = -(x + 3)^2 + 19$; (−3, 19); (0, 10); $a = -1$

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